# Effect of a Nonconstant $C_{m\alpha}$ on the Stability of Rolling Aircraft

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**This is an analytical study of the behavior of modern high-speed aircraft of inertially slender configurations in maneuvers involving large rates of roll. Inertia cross-coupling, as well as a linear variation of $C_{m\alpha}$ with angle of attack, are considered. The steady-state solutions of the nonlinear equations of motion, based on principal inertia axes, are studied to obtain useful information on the response behavior of the state variables during roll maneuvers. It is shown that, in addition to the critical values of aileron deflection that have been previously found to limit a steady-state roll with constant $C_{m\alpha}$, there can be two new critical values introduced by a linear decrease of $|C_{m\alpha}|$ with angle of attack. For aileron deflections near these critical values the response of the aircraft exhibits violent oscillations and dangerous peak loads due to the cross-coupled motion accompanying a roll maneuver. These critical values define a new range of aileron deflections in which no steady-state roll is possible.**

## Nomenclature

| | |
|---|---|
| $A$ | = aspect ratio = $b/\bar{c}$ |
| $B$ | = $(I_z - I_x)/I_y$ |
| $b$ | = wingspan |
| $C$ | = $(I_y - I_x)/I_z$ |
| $\bar{c}$ | = mean aerodynamic chord |
| $I_x, I_y, I_z$ | = moments of inertia relative to body axes |
| $i_x, i_y, i_z$ | = moments of inertia for principal inertia axes in nondimensional form (Table 1) |
| $L, M, N$ | = rolling, pitching, and yawing moments |
| $m$ | = mass of the aircraft |
| $m'$ | = $m_0 - 2m_l w_s + w_0 m_l$ |
| $m_0$ | = $-m_w(0)/B$ |
| $m_l$ | = $(\partial m_w/\partial w)/B$ |
| $n$ | = $n_v/C$ |
| $P, Q, R$ | = rates of roll, pitch, and yaw |
| $p, q, r$ | = nondimensionalized forms of $P$, $Q$, $R$ (Table 1) |
| $q_0$ | = initial rate of pitch of the principal inertia axis |
| $S$ | = wing area |
| $t$ | = time, s |
| $t^*$ | = $m/(\rho S V_0)$ |
| $U, V, W$ | = velocity components along the body axes |
| $V_0$ | = forward speed of the aircraft |
| $v$ | = $V/V_0$, angle of sideslip of the principal inertia axis |
| $w$ | = $W/V_0$, angle of attack of the principal inertia axis |
| $w_0$ | = initial angle of attack of the principal inertia axis |
| $X, Y, Z$ | = components of the aerodynamic force along the body axes |
| $\eta, \xi, \zeta$ | = deflection angles (in rad) of the elevator, aileron, and rudder |
| $\mu$ | = $m/(\rho S b/2)$, nondimensionalized mass (Table 1) |
| $\xi_0, \xi_{c1}, \xi_{c2}$ | = constant aileron deflection and its critical values |
| $\rho$ | = air density |
| $\tau$ | = nondimensionalized time (aero-seconds) |

*Operators*

| | |
|---|---|
| $(\dot{\ })$ | = $(d/dt)(\ )$ |
| D | = $(d/d\tau)$ |

*Subscripts*

| | |
|---|---|
| $s$ | = steady-state |

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## I. Introduction

THE design of modern high-speed aircraft has been associated both with a larger concentration of mass in the fuselage and with smaller wing aspect ratios than in more conventional configurations. When this type of aircraft (termed "inertially slender" by Pinsker[5]) is required to roll rapidly at moderate or high angles of attack, it experiences undesirable response characteristics that cannot be predicted by linear theory. For these roll maneuvers the equations of motion of an inertially slender aircraft can no longer be separated into two independent sets of longitudinal and lateral linear differential equations. Inertia cross-coupling also introduces nonlinearities into the equations of motion, thus leading to at least five nonlinear ordinary differential equations.

Recently the need has arisen to study the behavior of these equations analytically in order to determine the major characteristics of the response to control inputs, and to derive simple stability criteria that would be useful at the design stage. To achieve this goal the complicated equations of motion must be considerably simplified before analysis to provide a better physical understanding of the problem.

The first analytical study of rapidly rolling aircraft was given by Phillips[1] in 1948 when he studied the stability of an inertially slender aircraft rolling with a constant rate of roll. In his simple 4-degree-of-freedom analysis Phillips found the critical values of the roll rate at which the aircraft motion becomes unstable. Although Phillips' analysis was the first to explain the crucial effects of inertia cross-coupling, it failed to provide an order-of-magnitude estimate of the state variables in a cross-coupled rolling maneuver. Pinsker[2-6] showed, through simplified physical reasoning, that a fast-rolling slender aircraft, at moderate angles of attack, rolls about its longitudinal principal inertia axis. The resulting rolling oscillation is quite distinct from typical lateral modes of motion in conventional configurations. He also showed that under certain circumstances an aircraft with these extreme configurations may attain a sustained rolling motion called autorotation.

Several other authors[7-10] have studied this problem and Walsh,[7] by analyzing the possible steady states of the 5-degree-of-freedom equations of motion, showed that the longitudinal and lateral static stability derivatives have substantial effects on the steady-state values of the state variables during a rolling maneuver involving large rates of roll.

Since large values of angle of attack are reached during this type of maneuver, and since $C_{m\alpha}$ can then vary considerably with the angle of attack, it is logical to allow the variation of $C_{m\alpha}$ with $\alpha$ and use the method proposed by Walsh[7] to study the steady-state solutions.

## II. Equations of Motion

The studies of Phillips[1] and Pinsker[2-6] on the inertia cross-coupling during rolling maneuvers showed that one should use the principal axes through the center of gravity of the aircraft, and one could assume that the forward velocity $V_0$ remained constant and much larger than the sideslip velocity produced by the maneuver. Next, Thomas and Price[8] and Walsh[7] compared the orders of magnitude of the various terms left in the nonlinear equations of motion for a typical inertially slender aircraft and, by retaining only those terms necessary for a study of a response to a rolling maneuver, obtained the following:

$$\begin{aligned} &\mathrm{D}v - pw + r \approx 0 \\ &\mathrm{D}w + pv - q \approx z_w w \\ &\mathrm{D}p \approx l_v v + l_p p + l_\xi \xi \\ &\mathrm{D}q - Bpr \approx m_w w \\ &\mathrm{D}r + Cpq \approx n_v v \end{aligned} \tag{1}$$

The divisors used to nondimensionalize all of the terms in Eq. (1) are presented in Tables 1 and 2, which also indicates the equivalent NASA notation.

Using the data given by NASA[11-14] on a high-speed inertially slender aircraft it can be verified that these equations are satisfactory for analyzing high-speed roll maneuvers. However, the simplified Eq. (1) ceases to be valid whenever any of the following occur: 1) $m_w$ or $n_v$ become very small, a condition that can occur at very high angles of attack; or 2) $p$ becomes so small that the approximations involved in developing the preceding equations become invalid.

It is expected that these five nonlinear and coupled ordinary differential equations will give conservative results with respect to the stability of the motion, since the damping terms $n_r$ and $m_q$ have been neglected. With these simplifications these equations are now suitable for an analytical study of the steady-state solutions.

**Table 1 The nondimensional system**

| Dimensional quantity | Divisor | Nondimensional quantity |
|---|---|---|
| $t$ | $t^* = m/(\rho S V_0)$ | $\tau$ |
| $m$ | $\rho S(b/2)$ | $\mu$ |
| $P,Q,R$ | $(\rho S V_0)/m = 1/t^*$ | $p,q,r$ |
| $I_x,I_y,I_z$ | $m(b/2)^2$ | $i_x,i_y,i_z$ |

**Table 2 Stability derivatives and their relations to NASA notations[a]**

| NASA notations[15] | Divisors | As used in this paper |
|---|---|---|
| $C_{lp},C_{lr}$ | $i_x$ | $l_p,l_r$ |
| $C_{l\beta},C_{l,\delta a},C_{l,\delta r}$ | $i_x/\mu$ | $l_v,l_\xi,l_\zeta$ |
| $C_{mq}$ | $A^2 i_y$ | $m_q$ |
| $C_{m\alpha},C_{m,\delta e}$ | $A^2 i_y/\mu$ | $m_w,m_\eta$ |
| $C_{np},C_{nr}$ | $i_z$ | $n_p,n_r$ |
| $C_{n\beta},C_{n,\delta a},C_{n,\delta r}$ | $i_z/\mu$ | $n_v,n_\xi,n_\zeta$ |
| $C_{yp},C_{yr}$ | $2\mu$ | $y_p,y_r$ |
| $C_{y\beta},C_{y,\delta a},C_{z\alpha},C_{z,\delta e}$ | $2$ | $y_v,y_\xi,z_w,z_n$ |
| $C_{zq}$ | $2\mu A$ | $z_q$ |

[a]Should be evaluated relative to principal inertia axes before using them in the results of this paper. Example: $l_v = \mu C_{l,\beta}/i_x$.

## III. Study of the Possible Steady States

A number of important features of the dynamic response of an inertially slender aircraft in a rolling maneuver can be explained by the steady-state solutions. For example, the analysis of the steady-state solutions of the simplified equations of motion gives a first approximation to the critical rates of roll and the critical values of the initial angle of attack, at which the solution exhibits either divergence or a sudden jump from one steady state to another.

Phillips[1] constant-rate-of-roll analysis showed that the magnitudes of the longitudinal and lateral static stabilities had a direct bearing on the critical rates of roll. Walsh[7] found that aileron angles which produced rates of roll near the natural frequencies of the nonrolling aircraft resulted in undesirably large magnitudes of the sideslip, angle of attack, and rate of pitch, thus implying excessive accelerations and undesirable load peaks.

On the other hand, the longitudinal and lateral static stabilities (that is, $C_{m\alpha}$ and $C_{n\beta}$) diminish in magnitude as the angle of attack increases. Since inertia cross-coupled motions are frequently associated with large variations in the angle of attack, the most logical extension to the analysis is to include the variations of $C_{m\alpha}$ and $C_{n\beta}$ in Eq. (1).

Unlike all previous analyses,[1-10] which assumed constant values for all stability derivatives, $C_{m\alpha}$ (i.e., $m_w$ according to our notation) will now be assumed to vary with angle of attack. The analytical method proposed by Walsh[7] will then be used to study the characteristics of the steady states of the motion resulting from a constant aileron deflection. However, in order to be able to handle the equations analytically, only a linear variation of $C_{m\alpha}$ with the angle of attack will be assumed. The equations of motion now can be written as

$$\begin{aligned} &\mathrm{D}v = pw - r \\ &\mathrm{D}w = -pv + q + z_w(w - w_0) \\ &\mathrm{D}p = l_v v + l_p p + l_\xi \xi \\ &\mathrm{D}q = Bpr + m_w(w - w_0) \\ &\mathrm{D}r = -Cpq + n_v v \end{aligned} \tag{2}$$

where $w_0$ is the initial angle of attack of the principal inertia axis,

$$\xi = \begin{cases} 0 & (\tau \leq 0) \\ \xi_0 & (\tau > 0) \end{cases}$$

$m_w = m_w(0) + (\partial m_w/\partial w)\,w$, and the initial values of the other state variables are assumed to be zero.

Since the right-hand sides in Eq. (2) are all continuously differentiable functions of the state variables, the steady-state relations are given by

$$r_s = p_s w_s$$

$$q_s = p_s v_s - z_w(w_s - w_0)$$

$$l_v v_s + l_p p_s + l_\xi \xi_0 = 0$$

$$Bp_s r_s + (m_w)_s (w_s - w_0) = 0$$

$$-Cp_s q_s + n_v v_s = 0 \tag{3}$$

where

$$(m_w)_s = m_w(0) + (\partial m_w / \partial w) w_s$$

The above equations lead to:

$$p_s^2 = \frac{(m_0 - m_1 w_s)(w_s - w_0)}{w_s} \tag{4}$$

$$v_s = \frac{p_s z_w (w_s - w_0)}{(p_s^2 - n)} \tag{5}$$

$$q_s = \frac{n z_w (w_s - w_0)}{(p_s^2 - n)} \tag{6}$$

$$r_s = p_s w_s \tag{7}$$

$$l_v v_s + l_p p_s + l_\xi \xi_0 = 0 \tag{8}$$

where the constants

$$m_0 = -m_w(0)/B, \quad m_1 = (\partial m_w/\partial w)/B, \quad n = n_v/C$$

are assumed positive, which is usually the case.

The graphical method used in this study consists of plotting the curves of $v_s$, $p_s$, $q_s$, $r_s$, and $\xi_0$ versus $w_s$. Then, for a given value of $\xi_0$, it is possible to find the corresponding steady-state values of the angle of attack, $w_s$ (if they exist). These values of $w_s$ yield, from Eqs. (4) – (7), the steady-state values of other state variables (that is $p_s$, $q_s$, $r_s$, and $v_s$). It has been customary to consider the curve $\xi_0$ versus $p_s$ also, since this gives more physical and practical insight of the nature and consequences of the problem in hand, as shown by Walsh.[7] Figures 1 and 2 show the positive steady-state roll ($p>0$) produced by a negative aileron deflection ($\xi_0<0$), and the corresponding steady-state angle of attack ($w_s$), during autorotation in roll for both a constant $C_{m\alpha}$ ($m_1=0$) and a linearly decreasing $|C_{m\alpha}|$ ($m_1>0$). Figure 1 is for a positive initial angle of attack ($w_0=0.1$ rad), and Fig. 2 is for a negative initial angle of attack ($w_0=-0.05$ rad) for the inertially slender aircraft defined in Table 3. The dotted lines ($m_1=0$) are similar to Walsh's[7] solution since they are both for constant $C_{m\alpha}$.

## IV. Critical Values of Aileron Deflections

In Fig. 1 for $w_0=0.1$ it is seen that if $|C_{m\alpha}|$ decreases linearly ($m_1>0$) then one can obtain at most only two steady-state solutions; and there is a finite range of aileron deflections which has no steady-state solution. This critical range is due to the linear decrease of $|C_{m\alpha}|$ with angle of attack and is not present in the constant $C_{m\alpha}$ analysis of Walsh[7] or Thomas and Price.[8]

The aileron deflections for which no steady-state solution is possible are situated in the interval $\xi_{c1}<\xi_0<\xi_{c2}$, where $\xi_{c1}$ and $\xi_{c2}$ are the minimum and the maximum of the curve $-\xi_0(w_s)$, respectively. These extrema points are given by $w_s=w_{c1}$ and $w_s=w_{c2}$, respectively, where the latter are those solutions of $d\xi/dw_s=0$ for which the right-hand side of Eq. (4) becomes positive. These critical values of aileron deflection depend on $w_0$, $m_0$, $m_1$, $n$, and other aircraft parameters. They are seen to be associated with very large oscillations in all state variables with regard to the solutions of Eq. (2). The response to the application of an aileron deflection whose value is in the interval $[\xi_{c1}, \xi_{c2}]$ grows indefinitely because there is no steady-state solution possible for $\xi_0$ in that interval. The existence of these critical values of aileron deflection is a result (and perhaps the most important consequence) of the fact that the variation of $C_{m\alpha}$ with angle of attack is considered here. In the case of $m_1=0$ (constant $C_{m\alpha}$) this phenomenon was absent. Notice that this critical interval may contain those aileron deflections which resulted in stable response when $C_{m\alpha}$ was assumed constant. This divergence is clearly shown in the response of Fig. 3 when $m_1>0$. The value $\xi_0=-0.25$ in these calculations results in a stable response for constant $C_{m\alpha}$ ($m_1=0$), whereas the response for the case of a decreasing $|C_{m\alpha}|$ starts diverging at an angle of attack well below that for which $C_{m\alpha}$ becomes zero. [This occurs at $\tau\approx2.5$ (aeroseconds), while $C_{m\alpha}$ is still negative.] See also Fig. 1a, comparing the dashed lines ($C_{m\alpha}=$ const.) with the solid lines ($|C_{m\alpha}|$ decreasing with $\alpha$).

## V. Stability of the Possible Steady States

For an aileron deflection near critical, the solution to the simplified Eq. (2) exhibits large variations in all state variables. This is due to the proximity of an unstable equilibrium point to a stable one, thus leading to limit-cycle-type oscillations. The jump from one equilibrium point to another results in excessive accelerations, unacceptable to both the pilot and the aircraft.

The analysis of the stability in the large of the possible steady states is obviously prohibitive, both because of the nonlinearities introduced and because of the 5 degrees-of-freedom considered. Therefore, only stability in the small of the equations of motion Eq. (2), linearized around a steady state, is considered. The linearized equations of motion read

$$DX = AX \tag{9}$$

where

$$X' = (\delta v, \delta w, \delta p, \delta q, \delta r)$$

and

$$A = \begin{bmatrix} 0 & p_s & w_s & 0 & -1 \\ -p_s & z_w & -v_s & 1 & 0 \\ l_v & 0 & l_p & 0 & 0 \\ 0 & -Bm' & Br_s & 0 & Bp_s \\ C_n & 0 & -Cq_s & -Cp_s & 0 \end{bmatrix}$$

and where

$$m' = m_0 - 2m_1 w_s + w_0 m_1$$

The characteristic equation of this linearized system is given by

$$\det(\lambda I - A) = 0$$

which yields the following 5th-degree polynomial equation

$$\lambda^5 + E_4\lambda^4 + E_3\lambda^3 + E_2\lambda^2 + E_1\lambda + E_0 = 0 \tag{10}$$

where

$$E_4 = -(z_w + l_p)$$

$$E_3 = -l_v w_s + BCp_s^2 + p_s^2 + Bm' + Cn + l_p z_w$$

$$E_2 = l_v(p_s v_s + z_w w_s - Cq_s) - BCz_w p_s^2 - Cnz_w - l_p(BCp_s^2 + p_s^2 + Bm' + Cn)$$

$$E_1 = -l_v(Bp_s r_s + BCw_s p_s^2 + Bw_s m' + BCp_s r_s - Cz_w q_s) + BC(p_s^2 - m')(p_s^2 - n) + Cl_p z_w(Bp_s^2 + n)$$

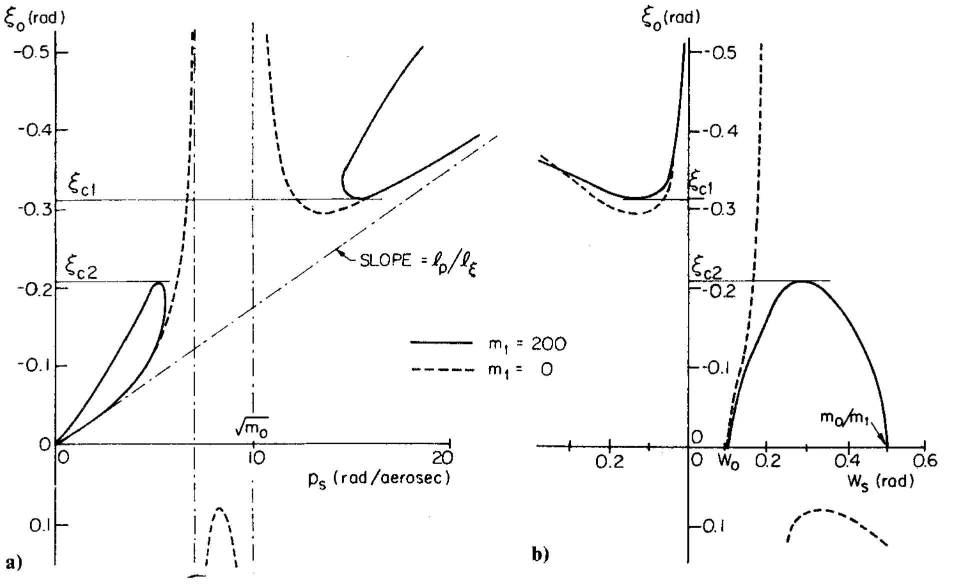


**Fig. 1 Steady positive rates of roll for a positive initial angle of attack ($w_0 = 0.1$).**

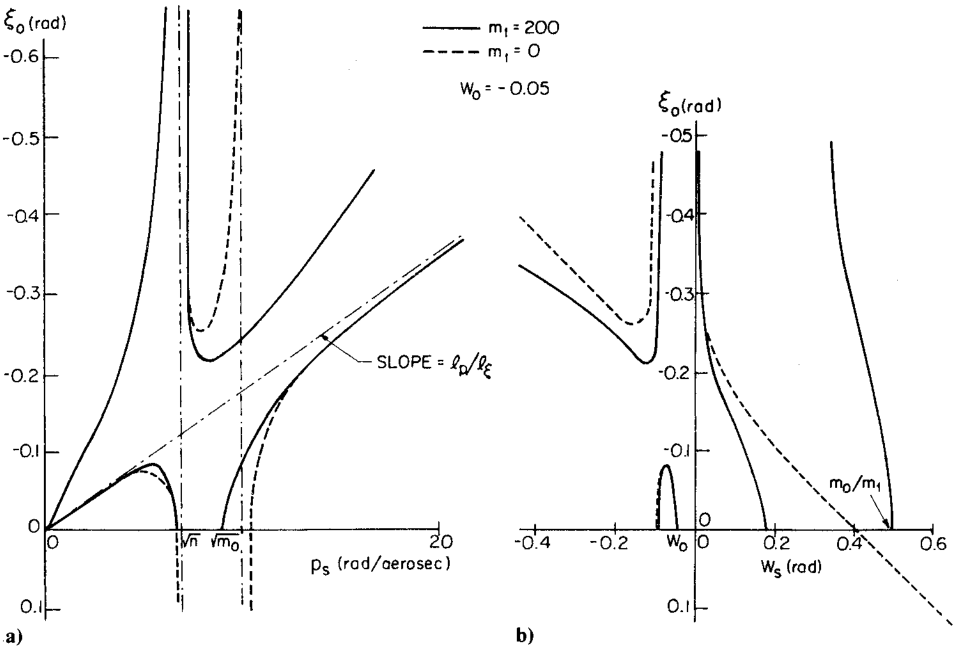


**Fig. 2 Steady positive rates of roll for a negative initial angle of attack ($w_0 = -0.05$).**

$$E_0 = BC\{ l_v [ (p_s^2 - m')(q_s + p_s v_s) + z_w (w_s p_s^2 + r_s p_s) ] - l_p (p_s^2 - m')(p_s^2 - n) \}$$

Complete stability analysis of the system, represented by the characteristic Eq. (10), requires the investigation of the signs of the Routh's test functions. The necessary and sufficient condition for the linear system to be asymptotically stable is that all the Routh's test functions be strictly positive. Obviously an analytical investigation of the signs of all the Routh's test functions is prohibitive in the general case because of the complexity of the above expressions. Fortunately, however, in almost all cases of practical interest, investigation of the sign of $E_0$ alone will be enough to provide useful information on the stability of the linearized Eq. (9).

Replacing $v_s$, $p_s$, $q_s$, and $r_s$ in the expression of $E_0$ by their relations as functions of $w_s$, from Eqs. (4-7), and after some manipulations it can be shown that

$$E_0 \leq 0 \text{ whenever } p_s \cdot \frac{d\xi_0}{dw_s} \leq 0$$

It results from the above condition that, for $p_s > 0$, the portions of the curve $-\xi_0(w_s)$ that have negative slope correspond to the unstable equilibrium states and that almost all parts of that curve with positive slopes correspond to the stable equilibrium states. Therefore, the extrema of the curve $-\xi_0(w_s)$, that is $\xi_{c1}$ and $\xi_{c2}$, are the critical values of aileron deflection, corresponding also to the critical rates of roll.

## VI. Conclusions

An analytical method has been discussed to study the response behavior of inertially slender aircraft to aileron deflections that produce large rates of roll. These maneuvers, being associated with a coupling between longitudinal and lateral motions of the aircraft, require the study of at least five coupled nonlinear ordinary differential equations.

The variation of the static longitudinal stability $C_{m\alpha}$ with the angle of attack is seen to introduce new phenomena in the response of the aircraft in roll. The most important of these is the existence of two critical aileron angles at which the responses exhibit large oscillations. These critical aileron values are in addition to those that produced unstable rates of

**Table 3 Flight conditions and aircraft parameters for inertially slender aircraft**

| | | | |
|---|---|---|---|
| Flight conditions | | Working coefficients (cont.) | |
| Altitude | $=30{,}000$ ft (9144 m) | $i_x$ | $=0.03$ |
| $V_0$ | $=700$ ft/s (213.36 m/s) | $i_y$ | $=0.19$ |
| Mach no. | $=0.7$ | $i_z$ | $=0.22$ |
| $\frac{1}{2}\rho V_0^2$ | $=217.9$ lb/ft$^2$ (10430 N/m$^2$) | $B$ | $=0.96$ |
| | | $C$ | $=0.727$ |
| | | $t^*$ | $=2.75$ |
| Aircraft characteristics | | Stability derivatives | |
| $m$ | $=1544$ slugs (22,530 kg) | $y_v$ | $=-0.4$ |
| $S$ | $=525$ ft$^2$ (48.77 m$^2$) | $z_w$ | $=-2.25$ |
| $b$ | $=63$ ft (19.20 m) | $l_v$ | $=-370$ |
| $\bar{c}$ | $=9$ ft (2.743 m) | $l_p$ | $=-6.$ |
| $I_x$ | $=53{,}000$ slug-ft$^2$ (71,860 kg-m$^2$) | $l_\xi$ | $=-340$ |
| $I_y$ | $=300{,}000$ slug-ft$^2$ (406,800 kg-m$^2$) | $l_\zeta$ | $=25.$ |
| $I_z$ | $=340{,}000$ slug-ft$^2$ (461,000 kg-m$^2$) | $m_w$ | $=-96.$ |
| Working coefficients | | $n_v$ | $=36.$ |
| $\mu$ | $=105.7$ | | |
| $A$ | $=\ 7.$ | | |

[a] The numerical examples chosen in this report correspond to the data on a modern supersonic slender-body aircraft provided by NASA.[11-14] The flight conditions and the aircraft parameters considered are as given in the above table.

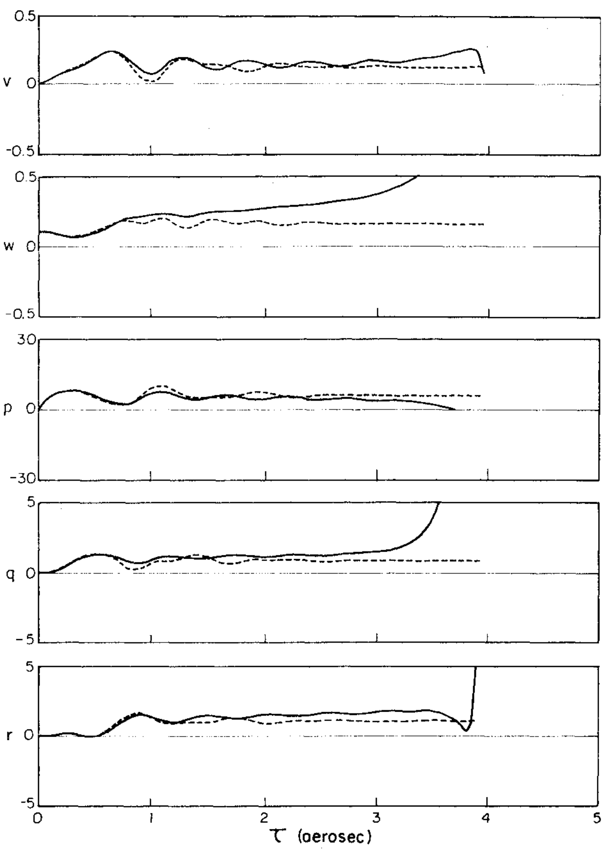


**Fig. 3 Response to aileron deflection $\xi_0 = -0.25$ with initial angle of attack $w_0 = 0.1$: (----), $m_l = 0$; (———), $m_l = 200$.**

roll in the constant $C_{m\alpha}$ analysis. All values of the aileron deflections between these two critical points produce a divergence in the response of the aircraft.

The graphical method discussed in the present paper provides some quantitative information on the behavior of the state variables during steady-state motion, as well as the stability of these steady states. It has been shown that the constant term $E_0$, of the characteristic Eq. (10), changes sign at the extrema points of the curve $\xi_0(w_s)$ or $\xi_0(p_s)$. This clearly leads to simple criteria regarding the stability of the aircraft response to a certain aileron deflection.

A study of the steady-state plots also confirms the following:

1) Aileron effectiveness is considerably reduced at high roll rates and at high angles of attack.

2) The behavior of the aircraft of inertially slender configurations maneuvering at high roll rates depends largely on the initial angle of attack of the principal inertia axis.

The same method used to show the effect of a linear decrease in $|C_{m\alpha}|$ with angle of attack in Eq. (4) and Figs. 1 and 2, with $m_l > 0$, can be used to show the effect of a decrease in $C_{n\beta}$ with the yaw angle. This should also show undesirable response behaviors for inertially slender aircraft as the initial side-slip velocity $(v_0)$ is increased. The response to a rolling pullout maneuver could also be investigated by including an initial pitching velocity $(q_0)$ in Eq. (2).

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## References


[1] Phillips, W. H., "Effects of Steady Rolling on Longitudinal and Directional Stability," NACA TN 1627, 1948.

[2] Pinsker, W.J.G., "Aileron Control of Small-Aspect-Ratio Aircraft; In Particular, Delta Aircraft," Aeronautics Research Council (A.R.C.), R&M 3188, 1953.

[3] Pinsker, W.J.G., "Charts of Peak Amplitudes in Incidence and Sideslip in Rolling Maneuvers Due to Inertia Cross-Coupling," A.R.C. R&M 3293, 1958.

[4] Pinsker, W.J.G., "Critical Flight Conditions and Loads Resulting from Inertia Cross-Coupling and Aerodynamic Stability Deficiencies," AGARD Report 107, 1957.

[5] Pinsker, W.J.G., "The Lateral Motion of Aircraft, and in Particular of Inertially Slender Configurations," A.R.C. R&M 3334, 1961.

[6] Pinsker, W.J.G., "The Theory and Practice of Inertia Cross-Coupling," *Aeronautical Journal, Royal Aeronautical Society*, Vol. 73, Aug. 1969, pp. 695-702.

[7] Walsh, G. R., "Forced Autorotation in the Rolling Motion of an Aeroplane," *The Aeronautical Quarterly*, Vol. 17, Aug. 1966, pp. 269-284.

[8] Thomas, H.H.B.M. and Price, P., "A Contribution to the Theory of Aircraft Response in Rolling Maneuvers Including Inertia Cross-Coupling Effects," A.R.C. R&M 3349, 1960.

[9] Gates, O. B. and Minka, K., "Note on a Criterion for Severity of Roll-Induced Instability," *Journal of Aeronautics and Space Science*, Vol. 26, May 1959, pp. 287-290.

[10] Rhoads, D. W. and Schuler, J. M., "A Theoretical and Experimental Study of Airplane Dynamics in Large-Disturbance Maneuvers," *Journal of Aeronautical Science*, Vol. 24, July 1957, pp. 507-527.

[11]Gilbert, W. P. and Libbey, C. E., "Investigation of an Automatic Spin-Prevention System for Fighter Airplanes," NASA TN D-6670, 1972.

[12]Ray, E. J. and Hollingsworth, E. G., "Subsonic Characteristics of a Twin-Jet Swept-Wing Fighter Model with Maneuvering Devices," NASA TN D-6921, 1973.

[13]Grafton, S. B. and Libbey, C. E., "Dynamic Stability Derivatives of a Twin-Jet Fighter Model for Angles of Attack from $-10°$ to $110°$," NASA TN D-6091, 1971.

[14]Greer, H. D., "Summary of Directional Divergence Characteristics of Several High-Performance Configurations," NASA TN D-6993, 1972.

[15]Etkin, B., *Dynamics of Atmospheric Flight*, Wiley, New York, 1972, pp. 175-181.